

# **Topics in Operator Theory**

## **Edited by Carl Pearcy**

### **Mathematical Surveys, Number 13**

The five articles in this volume are expository in nature, and they all deal with various aspects of the theory of bounded linear operators on Hilbert space. The volume is very timely, because in the last year or two great progress has been made on hard problems in this field, and thus operator theory today is a very exciting area of mathematical research. One particular problem on which considerable progress has been made recently is the invariant subspace problem. This is the question whether every bounded linear operator on a separable, infinite-dimensional, complex Hilbert space  $H$  has a nontrivial invariant subspace. Even though this problem remains unresolved, there are some operators  $T$  on  $H$  for which the structure of the lattice of all invariant subspaces of  $T$  is known, and the first article in this volume, "Invariant subspaces", by Donald Sarason, is devoted to a discussion of such operators. One of the interesting features of this lucid presentation is the interplay between operator theory and classical analysis.

The second article is entitled "Weighted shift operators and analytic function theory" and was written by Allen Shields. He has taken essentially all of the information presently known about weighted shift operators (with scalar weights) and incorporated it into this comprehensive article. A central theme of the exposition is the interaction between weighted shift operators and analytic function theory, and as an added bonus for the reader, the article contains a list of thirty-two interesting research problems.

The third article in the volume is a treatise entitled "A version of multiplicity theory" by Arlen Brown. The problem treated is how to decide when two normal operators are unitarily equivalent. (Unitary equivalence is the analog for operators of the concept of isomorphism for groups, rings, etc.) The unitary equivalence problem for arbitrary operators is exceedingly difficult, but the theory of spectral multiplicity, which can be approached in several different ways, furnishes a reasonable complete set of unitary invariants for normal operators. The author focuses attention on the concept of a spectral measure, and his clear presentation of this circle of ideas should lead to a better understanding of multiplicity theory by beginners and experts alike.

The fourth article in this volume, "Canonical models" by R. G. Douglas, is concerned with the theory of canonical models for operators on Hilbert space. The central underlying idea is that if  $T$  is any contraction operator on  $H$  (i.e., if the norm of  $T$  is at most 1), then there is a canonical construction that associates with  $T$  an operator  $M_T$  that is unitarily equivalent to  $T$ , called its "canonical model". One can therefore study  $T$  by studying  $M_T$  instead, and this theory has made significant progress in the past ten years. The author, who has contributed substantially to the geometrization of this theory, exposes in his article various important components of the theory, and thereby gives the reader much insight into its successes and failures.

The final article in this volume, "A survey of the Lomonosov technique in the theory of invariant subspaces" by Carl Pearcy and Allen Shields, is a survey of some new invariant-subspace theorems that resulted from the brilliant and elegant method of proof introduced by Victor Lomonosov early in 1973. Further study and refinement of this technique should lead to additional progress on the invariant subspace problem.

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**PROCEEDINGS OF SYMPOSIA IN APPLIED MATHEMATICS, Volume 20**

**The Influence of Computing on Mathematical Research and Education,**

*Edited by Joseph P. LaSalle*

This volume contains seven of the invited addresses and fourteen of the contributed papers that were presented at the joint American Mathematical Society and the Mathematical Association of America Conference on the Influence of Computing on Mathematical Research and Education held at the University of Montana, August 13—24, 1973.

The invited addresses were directed primarily to the influence of the computer on mathematical research and the applications of mathematics and secondarily on what this means for the teaching of mathematics and the education of mathematicians. The contributed papers describe more specifically some experiments in developing courses in mathematics with computing and algorithmic orientations and a few reports on computer influenced research.

The titles of the seven invited addresses and their authors follow:

*The Influence of Computing on Research in Number Theory* by D. H. Lehmer

*The Influence of Computers on Algebra* by Charles C. Sims

*Computational Probability and Statistics* by Ulf Grenander

*An Introduction to Some Current Research in Numerical Computational Complexity*  
by J. F. Traub

*Applied Mathematics and Computing* by Peter D. Lax

*The Unexpected Impact of Computers on Science and Mathematics* by Thomas E. Cheatham, Jr.

The titles of the fourteen contributed papers and their authors follow:

*Computational Complex Analysis* by Peter Henrici

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*The Integration of Computing and Mathematics at the Open University* by F. B. Lovis  
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*Real Time Computer Graphics Techniques in Geometry* by Thomas Banchoff and  
Charles Strauss

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*Dual Orthogonal Series: A Case Study of the Influence of Computing upon Mathematical  
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*Some Problems in Computational Probability* by Marcel F. Neuts

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*On Using the Electronic Analog Computer to Illustrate Mathematical Concepts* by Tyre  
A. Newton

*An Inexpensive Computer Assist in Teaching Large Enrollment Mathematics Courses*  
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*A new Computer Oriented (Algorithmic) Linear Algebra Course—Preliminary Report*  
by Robert Ducharme

*Computer Supplemented Business Oriented Mathematics* by Kenneth L. Hankerson and  
Gene A. Kemper

Only some college training in mathematics is needed to read most of the volume. It should be of some interest to high school teachers of mathematics.

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# INDEX TO MATHEMATICS OF COMPUTATION, 1943 — 1969

Edited by

**Yudell L. Luke, Jet Wimp and Wyman Fair**

462 + xviii pages; list price \$19.95; institutional member price \$14.96; individual member price \$9.97

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The INDEX TO MATHEMATICS OF COMPUTATION is a compilation, by author and by subject, of all material which has appeared in MATHEMATICS OF COMPUTATION and its predecessor, MATHEMATICAL TABLES AND OTHER AIDS TO COMPUTATION, during the years 1943 — 1969—twenty-three published volumes. The INDEX contains over 7,000 entries. This is an unusual compilation because of the unique character of the journal which not only publishes research papers, but also publishes reviews of material on mathematics of computation and a table errata section covering a number of other publications. In addition, an unpublished mathematical tables (UMT) file is maintained.

A new classification system, which was developed in 1969 by a committee chaired by Yudell Luke at the Midwest Research Institute, is used in the *subject classification index*. In this section, all articles, tables, reviews, etc. are classified. The classification scheme is designed as an indexing system for retrieval of information in MATHEMATICS OF COMPUTATION, and the present index contains classification numbers for all entries beginning with 1943.

The *author index* has been set up so that it gives bibliographical information on all of the items published in the journal. Information in this index includes title of article; translated title of books not in English, French, German, or Italian; title of book which is a collection of articles written by a number of authors; volume, year, and number of pages; publisher of a book; journal title of periodicals, information concerning translations; MATHEMATICAL REVIEWS numbers; subject classification. Items are listed alphabetically by author and chronologically under each author's entry. Each article is identified by both an ordering numeral and an identifying code in order that the reader may tell quickly whether the information listed concerns a primary research publication, a review of the work, errata to a table, or any of the other types of information covered by the journal. The following codes are used:

P = research paper

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T = table in UMT (unpublished mathematical table) file

E = table errata

Q = queries and replies

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These identifying codes appear in the *subject classification index*, also, to refer the reader to the *author index*.

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Smith, A. B. 1 PR, 3E

the reader knows immediately that under Smith's name in the *author index*, there will be listed both a research article on the subject of linear equations, written by Smith, and a review of the article. In addition, an errata to a table which Smith published in another journal will be listed.

The preparation and publication of the *subject classification index* was supported in part by a grant from the National Science Foundation (GN-691).

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The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor James H. Bramble, MATHEMATICS OF COMPUTATION, Center for Applied Mathematics, 275 Olin Hall, Cornell University, Ithaca, New York 14853.

# Mathematics of Computation

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